DECENTRALIZED AND SELF-CENTERED ESTIMATION ARCHITECTURE FOR FORMATION FLYING OF SPACECRAFT

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ABSTRACT – *Formation estimation methodologies for distributed spacecraft systems are formulated and analyzed. A generic form of the formation estimation problem is described by defining a common hardware configuration, observation graph, and feasible estimation topologies. The implementation tradeoffs are discussed. A new "self-centralized" formation estimation approach is introduced that is computationally efficient and that eliminates the need for algorithm restructuring and state reinitialization after formation reconfiguration or member failure. The self-centered filter can provide the optimal estimate for similar state representations via the general properties of the conditional mean. Several architectural attributes of the self-centered estimator are discussed and then compared to other potential approaches.*

KEYWORDS: Formation Flying, Decentralized Estimation, Formation Estimation

INTRODUCTION

Formation flying spacecraft refers to a set of spatially distributed spacecraft flying in formation with the capability of interacting and cooperating with one another. In order to perform a task, the formation must collectively and collaboratively act as a single unit. Formation flying spacecraft differ from constellation of spacecraft by their interactive and cooperative attributes. The estimation and control problems involve deciding how to share sensor information and control authorities between spacecraft.

Numerous NASA's future Earth and space science missions involve formation-flying spacecraft (e.g., Terrestrial Planet Finder (TPF), Terrestrial Planet Imager, Starlight, LISA) [1]. Maneuvering multiple spacecraft for these missions involves high precision alignments and synchronized translational and rotational movements. While some research has been done to understand the complexity involved [2-12] in achieving the precision coordination and control, reconfigurations, communication and station keeping for these missions, the formation estimation problem has not been adequately addressed.

In this paper, a systematic approach to formation estimation of a distributed spacecraft system is investigated. The estimation problem is treated in a generic sense and may be applied to any N spatially distributed spacecraft in a formation. Specific considerations such as formation reconfiguration and initialization will be addressed.

FORMATION ESTIMATOR DEVELOPMENT PROCESS

If a single spacecraft state estimator design is considered, a fixed form of the estimator algorithm can be chosen to handle most in-flight conditions such as various estimator mode switches (e.g. transition from sun point mode to earth point mode) or sensor failures. However, the problem is more complex when considering a multiple spacecraft case. Formation reconfiguration or formation member addition/failures can cause the need to change the formation estimator state definitions. From a practical point of view, it is highly desirable to have an estimator that does not require significant algorithm restructuring during those events. We now develop a systematic approach to the formation estimator design.

The building blocks that are used for the formation estimation consist of formation members and their associated hardware, and these will be used to define two types of links. To encapsulate the interchange and availability of information between spacecraft for estimation, we introduce the communication and observation graphs. The communication graph is a directed graph in which the vertices represent individual spacecraft and a directed edge, termed a *communication link*, indicates telemetry data is being communicated in the direction indicated. The observation graph is also a directed graph in which the vertices again represent spacecraft and the directed edge, termed an *observation link*, between vertices i and j indicates, quoting from our earlier work [11,12], that the spacecraft represented by vertex i has information flow from the spacecraft represented by vertex j of at least one of the following forms: 1) direct measurements, 2) communicated measurements and/or 3) communicated state estimates such that the relative states can be estimated. That is, an *observation link* is considered established from spacecraft i to j, if spacecraft i can reconstruct the relative position and velocity between spacecraft i and j. If the information is being communicated, the communication graph is used to determine the routing. An observation graph indicates availability of relative state information between spacecraft.

Once established, the graphs and the mission objectives provide grounds for defining formation state variables. Given formation state variables, a specific formation estimation topology must be chosen, for instance, for larger formations, it may be sensible to use only a local subset of all the available observation links. After fixing a topology, the estimation problem is clearly defined and hence the estimator itself be designed.

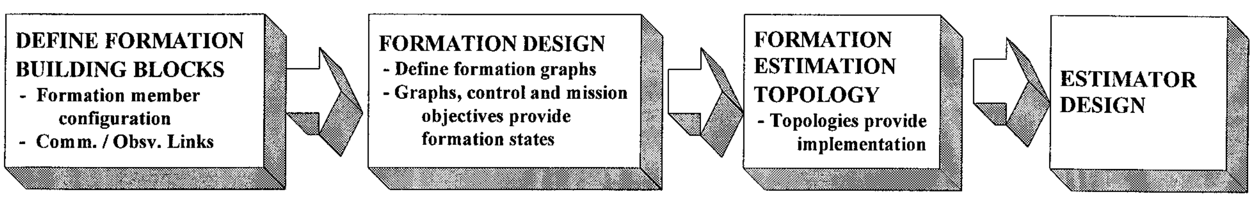

Figure 1: A Conceptual Formation Estimator Formulation Process

TYPICAL FORMATION FLYING SYSTEM HARDWARE

A common set of formation flying sensors consists of a 6 degrees-of-freedom (DOF) inertial sensors and relative position and velocity sensors. The inertial navigation sensors, such as 3 axis accelerometers, are necessary for a typical formation deployment phase wherein the separated spacecraft must self-navigate into an initial formation geometry without the aid of relative position sensors. Also, relative position sensors generally require an acquisition phase before they can provide measurements and can "go blind" due to a limited field-of-view. The inertial sensors are also necessary for formation member separation and recapture during an arbitrary formation reconfiguration.

In addition to the inertial sensors, relative position and velocity sensors are necessary since the inertial sensors alone cannot estimate the position of spacecraft without drifting over time. A combined package of inertial sensors and relative position/velocity sensors provide a good marriage of sensors as they compensate for one another's shortcomings. This particular sensor integration approach does not allow absolute inertial calibration of accelerometers, but calibrates only the relative biases. For many deep space missions, the inertial position knowledge is not needed. If an inertial calibration of an accelerometer is needed, other inertial reference position measurements such as the GPS can be incorporated.

Within a formation, spacecraft will be communicating measurements to each other, as the filter propagation process requires inertial sensor data from other spacecraft. In addition, measurements can be communicated to support the filter update. Therefore, the inter-spacecraft communication devices are an integral part of the formation estimation system.

A common reference frame is necessary to translate these communicated measurements into any particular spacecraft body frame. Inertial attitude sensors (gyros and star trackers), necessary for individual spacecraft attitude control, provide a common inertial reference frame to support the transformation of communicated data.

FORMATION OBSERVATION GRAPHS

As stated earlier, the observation graph is a directed graph in which the vertices represent individual spacecraft and a directed edge, termed an *observation link*, between vertices i and j indicates availability of relative state information between spacecraft. An *observation link* is considered established from spacecraft i to j, if the spacecraft i can reconstruct the relative position and velocity between spacecraft i and j. The equations underlying an observation links are now developed in detail.

Observation Link: Communication requirements for state propagation

Relative position and velocity between S/C_i and S/C_j can be propagated at S/C_i if certain inertial measurements are communicated from S/C_j. Consider the relative dynamics between the ith and jth spacecraft (S/C_i and S/C_j) as shown in Figure 2.

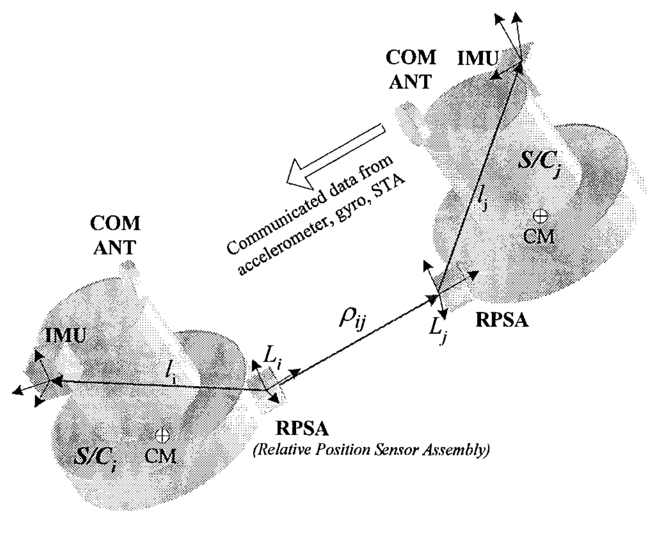

Figure 2: Observation link between spacecraft i and j

The relative motion between S/C_i and S/C_j at the Relative Position Sensor Assembly (RPSA) reference points, L_i and L_j, is:

$$\ddot{\rho}_{ij} = a_i^{JL} - M_{ij} a_j^{JL} = a_i^{JA} - M_{ij} a_j^{JA} + \omega_i^{JB} \times \left(\omega_i^{JB} \times l_i\right) - M_{ij}\left(\omega_j^{JB} \times \left(\omega_j^{JB} \times l_j\right)\right) + \alpha_i^{JB} \times l_i - M_{ij}\left(\alpha_j^{JB} \times l_j\right) \quad (1)$$

where a_i^{JL} and a_j^{JL} are the inertial accelerations of the points L_i and L_j, a_i^{JA} and a_j^{JA} are the inertial accelerations at the accelerometer locations, M_{ij} is the directional cosine matrix from jth spacecraft body frame to ith spacecraft body frame, $\omega_i^{JB}, \omega_j^{JB}$ and $\alpha_i^{JB}, \alpha_j^{JB}$ are the i-th and j-th spacecraft's body rate and angular acceleration, l_i and l_j are the vectors from the RPSA reference points to the corresponding accelerometer locations, and all vectors are expressed in body frame of the corresponding spacecraft. Replace a_i^{JA} and a_j^{JA} with the augmented accelerometer model of:

$$a_{pm}^{JA} = a_p^{JA} + b_p + v_p \qquad \text{and} \qquad \dot{b}_p = \eta_p \qquad \text{for } p = i, j \tag{2}$$

and define the relative accelerometer bias as:

$$b_{ij} = b_i - M_{ij} b_j \quad \text{where} \quad b_i = b_{io}, \quad \text{assumed to be constant} \tag{3}$$

where a_{pm}^{JA} is the accelerometer measurement, b_p is an accelerometer bias and v_p, η_p are white Gaussian noises. Then the (1) can be written as:

$$\begin{bmatrix} \dot{b}_{ij} \\ \dot{V}_{ij} \\ \dot{\rho}_{ij} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ M_{ij} & 0 & 0 \\ 0 & I_{3x3} & 0 \end{bmatrix} \begin{bmatrix} b_{ij} \\ V_{ij} \\ \rho_{ij} \end{bmatrix} + \begin{bmatrix} \eta \\ v \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \zeta + \psi \\ 0 \end{bmatrix} \tag{4}$$

where V_{ij} is the relative velocity between RPSA reference points on S/C_i and S/C_j, the *translational relative state vector* between spacecraft i and j is $x_{ij} = \begin{bmatrix} b_{ij}^T & V_{ij}^T & \rho_{ij}^T \end{bmatrix}^T$, and

$$\begin{gathered} v = -v_i + M_{ij} v_j \qquad \text{and} \qquad \zeta = a_{im}^{JA} - M_{ij} a_{jm}^{JA} + b_{io} \\ \psi = \omega_i^{JB} \times \left(\omega_i^{JB} \times l_i \right) + \alpha_i^{JB} \times l_i - M_{ij} \left[\left(\omega_j^{JB} \times \left(\omega_j^{JB} \times l_j \right) \right) + \alpha_j^{JB} \times l_j \right] \end{gathered} \tag{5}$$

As is seen from (4) and (5), in order to propagate the relative position and velocity between spacecraft i and j at spacecraft i, S/C_j must communicate the following information to S/C_i: M_{Jj} (directional cosine matrix from j-th spacecraft to inertial space J), $\omega_j^{JB}, \alpha_j^{JB}, l_j$ and a_{jm}^{JA}.

Observation Link: Measurement incorporation for state update

Accelerometers drift (see (2)); therefore, relative measurements are necessary to remove the integration error due to the drift. Measurement of ρ_{ij} or V_{ij} is needed for long-term accuracy in the estimator. Spacecraft i can obtain this measurement: 1) directly from its own RPSA, 2) from spacecraft j, if spacecraft j has measured the relative position (velocity) of spacecraft i with respect to itself, and/or 3) from a third spacecraft, if that spacecraft has measured the position of both spacecraft i and spacecraft j with respect to itself.

FORMATION ESTIMATION PROBLEM STATEMENT

Through processing of collected spacecraft sensor measurements, the formation estimator estimates the states needed by the formation control and guidance algorithms. As the spacecraft are assumed to have inertial attitude sensors (e.g. star trackers), we do not consider attitude estimation in this first formulation. That is, the formation attitude estimation problem is simpler in the sense that relative attitude can be determined from differencing inertial attitude estimates.

With this in mind, let x^F be the formation state vector; it incorporates the translational dynamics of the formation and all possible observation links. Note that this is a non-minimal state representation of the formation system. x^F is defined as:

$$x^F = \begin{bmatrix} x_{12}^T & x_{13}^T & \cdots & x_{1N}^T & x_{23}^T & x_{24}^T & \cdots & x_{2N}^T & \cdots & x_{(N-1)N}^T \end{bmatrix}^{\mathrm{T}} \tag{6}$$

Note that it accounts for redundancies of the form $\rho_{ij} = -\rho_{ji}$. All possible observation links are captured with *N(N-1)/2* translational relative state vectors, so x^F is of dimension 9 *N(N-1)/2.*

The estimation problem is to find the minimum variance estimate of x^F given the measurement history of Z:

$$\hat{x}^F = \underset{\hat{x}^F}{\arg\min}\ E\left\{\left(x^F - \hat{x}^F\right)\left(x^F - \hat{x}^F\right)^T\right\} \tag{7}$$

where ^ signifies an estimate and the conditioning on Z is assumed in the notation.

FORMATION ESTIMATION ARCHITECTURES

Formation estimation architectures (FEAs) can be divided into three categories: centralized, distributed and decentralized. Each FEA is discussed in detail.

Centralized Estimation Architecture

A FEA is centralized if there is a single formation-wide master filter that collects the sensor measurements, control inputs and configurations of all spacecraft and the master filter then estimates the formation state. Since the estimation is performed in a single filter, the cross correlation of state variables can be conveniently maintained. The entire covariance matrix can also be retained and propagated without being affected by spacecraft failures as long as the spacecraft hosting the master filter has not failed. Finally, as is made clear by comparison to other architectures subsequently, optimality is straightforward to ensure in the centralized FEA.

Some missions are well suited for the centralized FEA. For example, the composition and geometry of TPF allows a centralized FEA, as there are 4 nearly identical collector spacecraft and 1 combiner spacecraft. Optically, the combiner spacecraft plays a role of centralization. Therefore, using the combiner as the host of the centralized filter is reasonable. However, the centralized FEA has a number of drawbacks. First, a failure of the spacecraft that the master filter resides on would end estimation. Secondly, when a large-scale formation is considered, the dependence on a master can be functionally undesirable as it results in excessive communication and line-of-sight visibility requirements on the formation members. Finally, the computational burden is concentrated in one spacecraft processor.

Distributed Estimation Architecture

If each spacecraft has its own local estimator and provides estimates to a master filter that then combines the local state estimates into a formation state estimate, the FEA is said to be *distributed.* The master filter may specify the states that the local estimators estimate or it may simply collect what information is available.

This FEA enables distribution of the estimation computational burden to each spacecraft; however, this approach adds significant complexity to calculating the formation state correlation as follows. Since the local state estimates are collected instead of raw measurements, the estimates from two different spacecraft can be correlated (possibly due to sharing of same measurements between the two). If such correlation is not accounted for, the master filter's accuracy can be degraded, and for some nonlinear systems with an extended Kalman filter, it can cause instability. In addition to the complexity of state correlation treatment, the filter may have to restructure after a spacecraft failure depending on the subtasks assigned to the failed spacecraft.

Decentralized Estimation Architecture

The estimation architecture is considered *decentralized* if the formation lacks a formation-wide master filter. Each spacecraft estimates the subset of the formation state variables it requires for control with locally available sensor measurements and communicated data. In the distributed FEA, the spacecraft also had local estimators, but they passed their estimates to a master filter that then returned an estimate of the entire formation state vector; as a result every spacecraft has identical state estimates. In contrast, the decentralized architecture results in different estimates for different spacecraft. However, the estimates can be synchronized via communication devices if desired.

The main advantage of the decentralized architecture is that the formation estimation is robust against *any* single point failure of any formation member. Recall that the centralized FEA will fail if the master spacecraft fails. For the decentralized FEA, the impact of spacecraft failure is identical for all spacecraft and the problem does not propagate to the other members of the formation. Furthermore, if the filter algorithm is structured conveniently, then the local estimation process can continue uninterrupted. In such case, the local estimator gracefully tolerates a failure of a formation member.

SELF-CENTERED DECENTRALIZED ESTIMATION TOPOLOGY

A specialized form of the decentralized formation estimation architecture is to describe the formation system with the self-centered perspective as shown in Figure 3. Each spacecraft describes the relative state variables with respect to itself in a one-to-one manner. With this description, a failure of a formation member does not interrupt the estimation process and does not require a filter restructuring and initialization as is shown subsequently. A failed member or a disrupted inter-communication link can be gracefully tolerated by using the self-centered definition of the formation state vector. Due to this advantage we investigate this estimator methodology further.

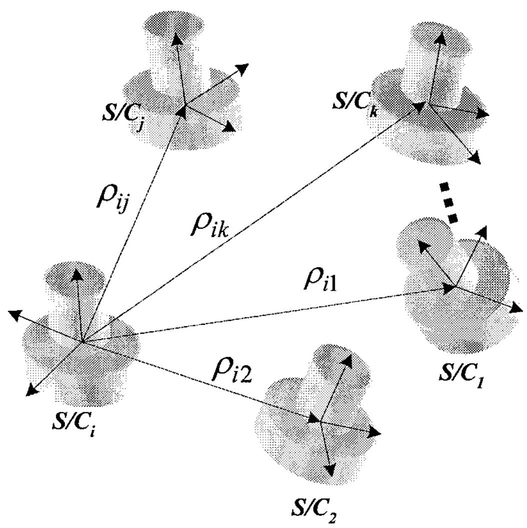

Figure 3: Self-centered representation of formation states

State and Measurement Models

The formation system of Figure 3 can be constructed by augmenting (4) for each inter-spacecraft translational relative state as follows. Write (4) as:

$$\dot{x}_{ij} = A_{ij}x_{ij} + w_{ij} + u_{ij} \qquad \text{for j=1, ... , N, and j} \neq \text{i} \tag{8}$$

Considering a particular realization of the formation state vector, $x^S = \begin{bmatrix} x_{i1}^T & x_{i2}^T & \cdots & x_{iN}^T \end{bmatrix}^T$, where the superscript "S" indicates "self-centered," the state space model of the self-centered system of Figure 3 is:

$$\dot{x}^S = \begin{bmatrix} A_{i1} & 0 & 0 & 0 \\ 0 & A_{i2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & A_{iN} \end{bmatrix} x^S + \begin{bmatrix} w_{i1} \\ w_{i2} \\ \vdots \\ w_{iN} \end{bmatrix} + \begin{bmatrix} u_{i1} \\ u_{i2} \\ \vdots \\ u_{iN} \end{bmatrix} = Ax^S + W + U \tag{9}$$

The RPSA measurements have the form:

$$\rho_{ij}^{m} = C_{ij}x^{S} + n_{ij} \tag{10}$$

where the observation matrix C_{ij} comes from the sensor model and n_{ij} is zero-mean, white, Gaussian sensor noise and the superscript "m" indicates a measurement. Note that particular sensor measurements may not always be available due to, for example, an obscuration of one spacecraft by another.

In addition to the direct measurement of (10), measurements can be communicated. For example, if ρ_{jk}^{m} is available (see Fig. 4) at spacecraft j, then spacecraft i treats the communicated measurement from spacecraft j as a direct measurement with the following observation matrix:

$$\rho_{jk} = \rho_{ik} - \rho_{ij} \quad \Rightarrow \quad \rho_{jk}^{m} = M_{ij}[0\cdots:0 \quad 0 \quad -I:0 \quad 0 \quad I:\cdots 0]x^{S} + n_{jk} = M_{ij}C_{jk}x^{S} + n_{jk} \tag{11}$$

where Mij is needed to rotate the measurement to spacecraft i's body frame.

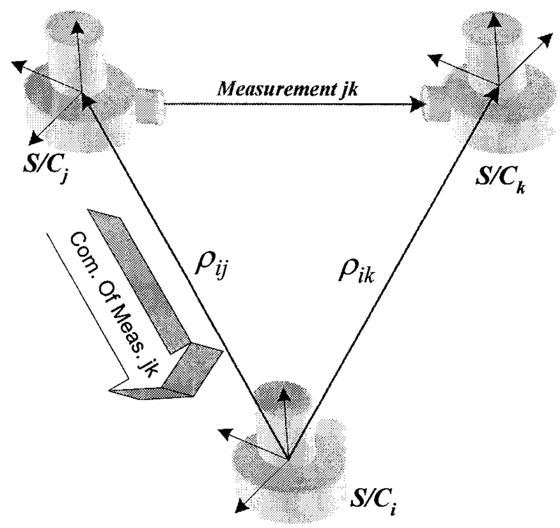

Fig. 4: Example of communicated RPSA sensor measurements

The self-centered estimation formulation described by (9), (10), and (11) is computationally efficient when compared to other decentralized, coupled topologies. First of all, the self-centered approach uses a minimal state representation. Then as shown by (9) the covariance propagation computational burden can be reduced by taking advantage of the block diagonal form of the A matrix.

Robustness to Formation Member Failure

As shown by the (9), the propagation of the relative translational state vector between one spacecraft and another is decoupled, assuming the plant noises, w_{ij}, are not correlated for all $i \neq j$, and neither are the controls, u_{ij}, $i \neq j$. Consider the following example using the model:

$$\begin{bmatrix} \dot{x}_{ij} \\ \dot{x}_{ik} \end{bmatrix} = \begin{bmatrix} A_{ij} & 0 \\ 0 & A_{ik} \end{bmatrix} \begin{bmatrix} x_{ij} \\ x_{ik} \end{bmatrix} + \begin{bmatrix} w_{ij} \\ w_{ik} \end{bmatrix} + \begin{bmatrix} u_{ij} \\ u_{ik} \end{bmatrix} = Ax^{S} + W + U \tag{12}$$

Prior to (condition 1) and after (condition 2) the failure of spacecraft k, the measurements are given by:

$$Z_1 = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_{ij} \\ x_{ik} \end{bmatrix} + n_1 \quad \text{and} \quad Z_2 = \begin{bmatrix} C_3 & 0 \end{bmatrix} \begin{bmatrix} x_{ij} \\ x_{ik} \end{bmatrix} + n_2 \tag{13}$$

Assuming a Kalman filter, we expand the governing Riccati equation for both conditions. For condition 1:

$$\dot{P} = AP + PA^{T} + Q - PC^{T}R^{-1}CP \qquad \Rightarrow$$

$$\begin{bmatrix}\dot{P}_{11} & \dot{P}_{12}\\ \dot{P}_{21} & \dot{P}_{22}\end{bmatrix}=\begin{bmatrix}A_{ij} & 0\\ 0 & A_{ik}\end{bmatrix}\begin{bmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{bmatrix}+\begin{bmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{bmatrix}\begin{bmatrix}A_{ij}^{T} & 0\\ 0 & A_{ik}^{T}\end{bmatrix}+\begin{bmatrix}Q_1 & 0\\ 0 & Q_2\end{bmatrix}-\begin{bmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{bmatrix}\begin{bmatrix}C_1^T\\ C_2^T\end{bmatrix}R_1^{-1}\begin{bmatrix}C_1 & C_2\end{bmatrix}\begin{bmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{bmatrix}$$

$$=\begin{bmatrix}A_{ij}P_{11}+P_{11}A_{ij}^{T}+Q_1 & A_{ij}P_{12}+P_{12}A_{ik}^{T}\\ A_{ik}P_{21}+P_{21}A_{ij}^{T} & A_{ik}P_{22}+P_{22}A_{ik}^{T}+Q_2\end{bmatrix}$$

$$-\begin{bmatrix}\left(P_{11}C_1^T+P_{12}C_2^T\right)R_1^{-1}\left(C_1P_{11}+C_2P_{21}\right) & \left(P_{11}C_1^T+P_{12}C_2^T\right)R_1^{-1}\left(C_1P_{12}+C_2P_{22}\right)\\ \left(P_{21}C_1^T+P_{22}C_2^T\right)R_1^{-1}\left(C_1P_{11}+C_2P_{21}\right) & \left(P_{21}C_1^T+P_{22}C_2^T\right)R_1^{-1}\left(C_1P_{12}+C_2P_{22}\right)\end{bmatrix} \tag{13}$$

For condition 2:

$$\begin{bmatrix}\dot{P}_{11} & \dot{P}_{12}\\ \dot{P}_{21} & \dot{P}_{22}\end{bmatrix}=\begin{bmatrix}A_{ij}P_{11}+P_{11}A_{ij}^{T}+Q_1 & A_{ij}P_{12}+P_{12}A_{ik}^{T}\\ A_{ik}P_{21}+P_{21}A_{ij}^{T} & A_{ik}P_{22}+P_{22}A_{ik}^{T}+Q_2\end{bmatrix}-\begin{bmatrix}\left(P_{11}C_1^T\right)R_2^{-1}\left(C_1P_{11}\right) & \left(P_{11}C_1^T\right)R_2^{-1}\left(C_1P_{12}\right)\\ \left(P_{21}C_1^T\right)R_2^{-1}\left(C_1P_{11}\right) & \left(P_{21}C_1^T\right)R_2^{-1}\left(C_1P_{12}\right)\end{bmatrix} \tag{14}$$

For condition 2, we have: $\dot{P}_{11}=A_{ij}P_{11}+P_{11}A_{ij}^{T}+Q_1-\left(P_{11}C_1^T\right)R_2^{-1}\left(C_1P_{11}\right)$ and so P_{11} equation is decoupled from P_{12}, P_{21} or P_{22}. This can be generalized if A_{ij} is replaced with a large square matrix (so conveniently to augment the upper block-diagonal system of healthy spacecraft). Consequently, a failed formation member can be gracefully treated within the filter without any filter state restructuring.

Preservation of Filter Optimality during a Linear State Space Transformation

In addition, optimal estimates for similar state descriptions are obtained by linear transformation of the self-centered state estimate. Let's consider a self-centered formation system of (from (9), (10) and (11)):

$$\begin{aligned}\dot{x}^S &= Ax^S+W+U\\ z &= Cx^S+n\end{aligned} \tag{15}$$

where W and n are the process and sensor noises, and z is the sensor measurement. The Kalman filter designed for (15) provides optimal solution of:

$$\hat{x}^S{}_{opt}=\arg\min_{\hat{x}} E\left\{\left(x^S-\hat{x}^S\right)^T\left(x^S-\hat{x}^S\right)\right\} \tag{16}$$

where the expectation E is conditioned on measurements Z. Consider a linear transformation of state x^S given by:

$$y=Mx^S \tag{17}$$

Proposition: if M invertible then $\hat{y}=M\hat{x}^S{}_{opt}$ is the solution to

$$\hat{y}_{opt}=\arg\min_{\hat{y}} E\left\{(y-\hat{y})^T(y-\hat{y})\right\} \tag{18}$$

Proof: Substituting (17) into the right hand side of (18)

$$E\left\{(y-\hat{y})^T(y-\hat{y})\right\}=E\left\{\left(x^S-M^{-1}\hat{y}\right)^T M^TM\left(x^S-M^{-1}\hat{y}\right)\right\} \tag{19}$$

Define $G=M^TM$, which is symmetric, to find the minimum we first take the derivative:

$$\tfrac{\partial}{\partial\hat{x}}E\left\{\left(x^S-M^{-1}\hat{y}\right)^T G\left(x^S-M^{-1}\hat{y}\right)\right\}=\tfrac{\partial}{\partial\hat{x}}E\left\{x^{S^T}Gx^S-\hat{y}^T\left(M^{-1}\right)^TGx^S-x^{S^T}GM^{-1}\hat{y}+\hat{y}^T\left(M^{-1}\right)^TGM^{-1}\hat{y}\right\} \tag{20}$$

$$= E\left\{-\left(M^{-1}\right)^T\left(Gx^S - G^T x^S\right) + \left(M^{-1}\right)^T GM^{-1}\hat{y} + \left(M^{-1}\right)^T G^T M^{-1}\hat{y}\right\}$$

Setting this result equal to zero yields

$$E\left\{-2\left(M^{-1}\right)^T Gx^S + 2\left(M^{-1}\right)^T GM^{-1}\hat{y}\right\} = 0 \tag{21}$$

Then,

$$\hat{y}_{opt} = \left(\left(M^{-1}\right)^T GM^{-1}\right)^{-1}\left(M^{-1}\right)^T GE\left\{x^S\right\} = M\hat{x}^S{}_{opt} \tag{22}$$

Again, this condition is valid for any M invertible. Consequently the optimal estimate $\hat{x}$ of the self-centered filter can provide the optimal estimate for $\hat{y}$ without recourse to another estimator.

CONCLUSION

Formation estimation methodologies for distributed spacecraft systems are formulated and analyzed. In particular, self-centered formation estimation approach is developed. The resulting self-centered approach is a computationally efficient algorithm, and the filter will operate without a need for restructuring of the state under formation member failures or formation reconfigurations. It is shown that the self-centered filter provides optimal estimates under linear nonsingular formation state transformations.

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